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A New Semi-analytical Modeling of Steam-Assisted Gravity Drainage in Heavy Oil Reservoirs

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Abstract

Thermal recovery by steam injection has proven to be an effective means of recovering heavy oil. Forecasts of reservoir response to the application of steam are necessary before starting a steam drive project. Thermal numerical models are available to provide forecasts. However, these models are expensive and consume a great deal of computer time. An alternative to numerical modeling is to use a semi-analytical model. The objective of current study was to investigate thermal applications of horizontal wells for displacement and gravity drainage processes using analytical modeling as well as reservoir simulation. The main novelties presented in the paper are entailed as; a) the transient temperature distribution ahead of the moving oil-steam interface is formulated, instead of classic assumption of quasi-steady distribution, b) New drainage oil rate calculation, c) Estimation of interface position while advancing into the reservoir towards the pattern boundaries, d) Introduction of new equations to place the interface curves into the reservoir, e) New "Type-Curves" are set up for approximating the interface velocity while propagating beyond the horizontal wells to the side boundaries, f) introducing an indirect formulations for evaluating the oil-steam interface velocity into the reservoir. Many cases were run on the classic approaches and proposed approach for comparison and verification purposes. The proposed methodology outperforms the previous and most recent models in view of precision and consistency.

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1. Introduction

In one decade, SAGD process has turned out to be the most promising strategy to develop huge heavy oil and bitumen accumulations (Butler et al (1980), Butler et al (1981), Aguilera et al (1991)). Like the conventional thermal processes (Butler et al (1980), Aguilera et al (1991), Edmunds (1999)), this method aims at reducing oil viscosity by increasing the temperature. In the SAGD process, this is achieved by drilling a pair of horizontal wells. Typically, the two horizontal drains are located at short distance one above the other, as shown in Figure 1.

Figure 1- SAGD Principle, (courtesy of McDaniel)

Steam is injected into the upper well and hot fluids are produced from the lower well. This progressively creates a chamber, which develops by condensing steam at the chamber boundary and giving latent energy to the surrounding reservoir. Heated oil and water are drained by gravity along the chamber walls towards the production well (Butler (1998)). Stable gravity displacement is particularly important to reach a favorable energy balance. In SAGD, the heated oil remains always in contact with the heated region, as it gets drained along the sidewalls of the steam chamber (Nasr et al (1999)). Thus, energy losses from heated oil, which has not been produced, are minimized.

According to Butler's original model (Butler et al (1998)), the drainage volumetric rate per one meter of the well length is determined by the height of steam chamber, as shown in Figure 2; the reservoir effective permeability (k), the gravity acceleration constant (g), the thermal diffusivity of reservoir (α), porosity (φ), displaceable oil saturation (ΔS_o), kinematic oil viscosity at steam temperature (ν_s), viscosity constant (m), and the model's constant C :

$$q = 2\sqrt{\frac{Ckg\ \alpha\phi\Delta s_o\left(h-y\right)}{m\,\upsilon_s}} \qquad (1)$$

Figure 2- Schema of SAGD process

This equation has been derived with considering some simplifying assumptions. Almost all the researchers have assumed that complete steam override occurs upon steam injection and that oil is heated from top to bottom due to conduction solely.

In the present work the transient temperature distribution ahead of the moving interface into the cold region is formulated. The relationship between the temperature distribution ahead of the moving interface and the oil rate due to SAGD production system is then derived. After doing a material balance formulation for the drained region based on a new method, the position of oil-steam interface is presented. Then, the interface positions into the half-width of reservoir based on Butler and Stephens' formula (1981) along with TANDRAIN theorem is formulated. By applying the new defined dimensionless groups and taking advantage of TANDRAIN phenomenon, the recovery factor of Butler et al (1981) is derived and compared to that of the proposed scheme. Likewise, the interface positions into the half-width of reservoir based on the proposed method along with TANDRAIN theorem is formulated and the recovery factor based on such new method is calculated afterwards. Also, a model which is simulated by a thermal simulator is described in detail and its results are then compared to those of the proposed method. Finally, a procedure to produce sets of type-curves in order to obtain a rough estimation of average interface velocity and interface velocity number is proposed.

2. Analytical Modeling of SAGD

Figure 3- SAGD production system

2.1 Temperature distribution – Consider a small section of a mature SAGD operation as depicted in Figure 3. At the steam-oil interface, steam condenses and heat is liberated. A thermal gradient is established via conduction between the steam temperature at the interface and the original reservoir temperature. As liquid drains via gravity out of differential element, steam moves in to replace the liquid. Consequently the interface moves at a certain velocity perpendicular to the oil-steam interface.

The governing equation for heat flow into the cold region via unsteady conduction heat transfer may be read as below:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

Initial and boundary conditions (Pooladi-darvish et al (1994)):

$$\begin{cases} t = 0,\ x \geq 0 : T = T_R \\ t > 0,\ x = \int_0^t U(\tau)d\tau : T = T_S \\ t > 0,\ x \to \infty : T = T_R \end{cases} \tag{3}$$

A new coordinate system is defined to avoid working with a moving boundary problem that travels with the interface (Pooladi-darvish et al (1994)).

$$\xi = x - \int_0^t U(\tau)d\tau \tag{4}$$

Where $U(t)$ is the interface velocity in the direction of ξ. This transformation fixes the moving interface at $\xi = 0$ for all time (Pooladi-darvish et al (1994)).

Using the following standard relationships

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial \xi}\left(\frac{\partial T}{\partial x}\right).\frac{\partial \xi}{\partial x} = \frac{\partial^2 T}{\partial \xi^2} \tag{5}$$

$$\left.\frac{\partial T}{\partial t}\right|_x = \frac{\partial}{\partial t}T(t,\xi) = \frac{\partial T}{\partial t}.\frac{\partial t}{\partial t} + \frac{\partial T}{\partial \xi}.\frac{\partial \xi}{\partial t} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial \xi}.(-U) \tag{6}$$

And substituting along with rearranging we may have (Pooladi-darvish et al (1994));

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{U}{\alpha}\frac{\partial T}{\partial \xi} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{7}$$

Defining the dimensionless groups and transforming the heat transfer equation into a normalized and dimensionless form enables us to work on analytical modeling with much more confidence. To do so, let us define them;

$$\theta = \frac{T - T_R}{T_s - T_R} \quad , \quad \tau = \frac{\alpha t}{h^2} \quad , \quad \zeta = \frac{\xi}{h} \quad , \quad N = \frac{Uh}{\alpha}$$

Where the parameter *N* may be called as “interface velocity number”. After all, the dimensionless heat transfer equation in a moving boundary problem such as SAGD can be found as;

$$\frac{\partial^2 \theta}{\partial \zeta^2} + N\frac{\partial \theta}{\partial \zeta} = \frac{\partial \theta}{\partial \tau} \tag{8}$$

For solving such equation there may be many mathematical methods, though here the method of Laplace transforms has been employed. Therefore;

$$\Theta_{\zeta\zeta} + N\Theta_{\zeta} = \Theta_{\tau} = L[\theta_{\tau}] = s\Theta - \theta(\zeta,0) = s\Theta - 0 = s\Theta \tag{9}$$

This is the Laplace transform of the transient heat equation and can be solved (in s-domain) analytically by applying the boundary conditions as below;

$$\Theta(\zeta, s) = \frac{e^{-\frac{N+\sqrt{N^2+4s}}{2}\zeta}}{s} \tag{10}$$

The inverse of this transform can be obtained through the use of some simple general theorems, that is;

$$\theta(\zeta,\tau) = \frac{1}{2}\left[e^{-N\varsigma}.erfc\left(\frac{\varsigma}{2\sqrt{\tau}} - \frac{N\sqrt{\tau}}{2}\right) + erfc\left(\frac{\varsigma}{2\sqrt{\tau}} + \frac{N\sqrt{\tau}}{2}\right)\right]^m \tag{11}$$

This is the transient temperature distribution ahead of the moving interface into the cold region, with the initial and boundary conditions of (3).

In the existing works that deal with analytical modeling of SAGD process, the temperature distribution is assume to be quasi-steady state and also the interface temperature remains constant. Therefore, the boundary condition of interface should be modified as the following:

$$\begin{cases} t = 0, \varsigma > 0 : \theta = 0 \\ t > 0, \varsigma = 0 : \theta = \theta_i \\ t > 0, \varsigma \to \infty : \theta = 0 \end{cases} \tag{12}$$

Where, $\theta_i = \dfrac{T_i - T_R}{T_s - T_R}$

Now, for any specified dimensionless time and any specified dimensionless distance, normalized temperature ahead of the moving interface may be expressed in terms of some dimensionless variables. The result has been obtained by solving the partial differential equation (eq. 8) over the initial and boundary conditions 12:

$$\theta(\zeta,\tau) = \frac{\theta_i}{2}\left[e^{-N\varsigma}.erfc\left(\frac{\varsigma}{2\sqrt{\tau}} - \frac{N\sqrt{\tau}}{2}\right) + erfc\left(\frac{\varsigma}{2\sqrt{\tau}} + \frac{N\sqrt{\tau}}{2}\right)\right]^m \tag{13}$$

This transient temperature distribution would have been more useful than that of Butler et al (1981), why the temperature time dependency has not been ignored. We may get to the point easily as soon as we formulate the oil recovery factor and compare the results with those of Butler et al (1981) in the headway.

2.2 Oil rate due to gravity drainage

This section is connected with the previous one which dealt with the temperature distribution in a SAGD problem. We are seeking here for a way by which we may recognize the relation between the temperature distribution ahead of the moving interface and the oil rate due to SAGD production system.

Applying Darcy's law and considering schema in figure 3 oil rate is approximated as (Aguilera et al (1991));

$$dq = \frac{k(\rho_o - \rho_g) g \sin\widehat{\varphi}}{\mu} d\xi \tag{14}$$

Since $\rho_o - \rho_g \cong \rho_o$, we obtain from equation 14 with integration over the entire length;

$$q = kg \sin\hat{\varphi} \int_0^{\infty} \frac{d\xi}{\upsilon} \tag{15}$$

υ is a fluid property which is a function of temperature, may be determined by using an equation of state (EOS) defining its dependence upon temperature. Here we use the equation suggested by Butler et al (1981) which has been used in the development of the SAGD theory.

$$\frac{\upsilon_s}{\upsilon} = \left(\frac{T - T_R}{T_s - T_R} \right)^m = \theta^m \tag{16}$$

So that oil rate is written as;

$$q = \frac{kg \sin\hat{\varphi}}{\upsilon_s} \int_0^{\infty} \theta^m d\xi \tag{17}$$

Now, we may use the transient temperature distribution of 13 to predict the rate of drainage to a horizontal well located at the ordinate "*y*" above the reservoir base.

$$q = \sqrt{\frac{2kg\alpha N\phi \Delta s_o (h-y)}{\upsilon_{os}}} \sqrt{\frac{\theta^m{}_i}{2^m} \int_0^{\infty} \left[e^{-N\varsigma}.erfc\left(\frac{\varsigma}{2\sqrt{\tau}} - \frac{N\sqrt{\tau}}{2} \right) + erfc\left(\frac{\varsigma}{2\sqrt{\tau}} + \frac{N\sqrt{\tau}}{2} \right) \right]^m d\varsigma} \tag{18}$$

This equation calculates the drainage rate for just one side of the reservoir. Therefore for the entire reservoir we should multiply this by 2.

In this equation "*k*" is effective permeability to oil flow. Therefore we should have the amount of k_{ro} that Butler and Stephen (1980) have assigned it for the sake of convenience as 0.4 as an average measure. It cannot be indeed calculated explicitly, so we should either guess a value being sound enough to cover the problem wholly or acquire it by using some nonlinear regression manipulations. For now, we consider it as a guess like that has been allocated by Butler and Stephen (1980). Further, we may rearrange the equation 18 as below:

$$q = 2.9922 \times 10^{-4} \sqrt{\frac{\sqrt{k_x k_z}\, g \alpha \phi \Delta s_o (h - y)}{m \upsilon_{os}}} \times q_D \tag{19}$$

$$q_D = \sqrt{\frac{m k_{ro} N \theta^m{}_i}{2^{m-1}} \int_0^\infty \left[e^{-N\varsigma} . erfc\left(\frac{\varsigma}{2\sqrt{\tau}} - \frac{N\sqrt{\tau}}{2} \right) + erfc\left(\frac{\varsigma}{2\sqrt{\tau}} + \frac{N\sqrt{\tau}}{2} \right) \right]^m d\varsigma} \tag{20}$$

Where k_{ro} is 0.4, (Butler et al (1980)). $\sqrt{k_x k_z}$ is an estimation for effective permeability to oil (md), g is acceleration due to gravity (m/s2), α is the thermal diffusivity of the reservoir material in (m^2/day), Δs_o is the displaceable oil saturation which is the difference between initial oil saturation and residual oil saturation (dimensionless), h is the vertical height over which drainage is occurring (m), y is the ordinate of a point on the interface over which the heated fluid is passing (m), υ_{os} is the kinematic viscosity of oil at steam temperature (cp.m^3/kg), and q is the oil drainage rate (m^3/day per one meter of horizontal production well) .

In this paper we have been seeking specially for two purposes; (1) Calculating the oil drainage rate including the effect of transient heat conduction effects, (2) Positioning the oil-steam interface as it advances to the reservoir boundaries beyond the wells.

Equations 28 and 29 serve our purpose to attain the first goal. For the second goal the following calculations have been made.

The drainage oil rate via SAGD that is suggested by Butler and Stephens (1980) is:

$$q = 2.9922 \times 10^{-4} \sqrt{\frac{2\sqrt{k_x k_z}\, g \alpha \phi \Delta s_o (h - y)}{m \upsilon_{os}}} \tag{21}$$

2.3 Steam-Oil Interface Positioning – The proposed scheme

Doing a material balance formulation for the drained region in an infinitesimal time step, we may obtain this (Aguilera et al (1991)):

$$\left[\frac{\partial q}{\partial x} \right]_t = \phi \Delta s_o \left(\partial y \big/ \partial t \right)_x \tag{22}$$

This expression accounts for the changing dimension of the steam zone as it expands at different rates vertically downward and horizontally across. Considering this equation and doing some mathematical manipulations the horizontal velocity at the interface is as follows:

$$(\partial x / \partial t)_y = \frac{(\partial y / \partial t)_x}{(\partial y / \partial x)_t} \tag{23}$$

Combining the two former equations we may have:

$$(\partial x / \partial t)_y = \frac{(\partial q / \partial t)_x}{\phi \Delta s_o (\partial y / \partial x)_t} \tag{24}$$

$$(\partial x / \partial t)_y = \frac{(\partial q / \partial y)_t}{\phi \Delta s_o} \tag{25}$$

Taking the partial derivative with respect to *y* in equation 19 and placing it in equation 25 results in:

$$(\partial x / \partial t)_y = 2.9922 \times 10^{-4} \sqrt{\frac{\sqrt{k_x k_z}\, g \alpha}{m \upsilon_{os} \phi \Delta s_o (h-y)}} \times \frac{q_D}{2} \tag{26}$$

Like the assumption that the steam chamber is initially a vertical plane above the well, the horizontal displacement *x* is given as a function of time *t* and height *y* by the relationship:

$$x = \left(2.9922 \times 10^{-4} \sqrt{\frac{\sqrt{k_x k_z}\, g \alpha}{m \upsilon_{os} \phi \Delta s_o (h-y)}} \times \frac{q_D}{2} \right) t \tag{27}$$

This may also be solved for *y* which results:

$$y = h - (\frac{q_D^2}{4}) \frac{(2.9922 \times 10^{-4})^2 \times \sqrt{k_x k_z}\, g \alpha}{m \upsilon_{os} \phi \Delta s_o} \left(\frac{t}{x} \right)^2 \tag{28}$$

These mathematical arrangements have been done before by Butler et al (1981), however it is modified here by both inserting the parameters which serve for the transient temperature distribution as well as defining the novel dimensionless groups. The schema in figure 3 depicts a typical interface which tends to progress away from the wells to the side boundaries. If the half-width of the reservoir is *w* and the height is *h*, we may define some dimensionless variables:

$$Y = \frac{y}{h} \quad , \quad X = \frac{x}{w} \quad , \quad t_D^* = q_D t_D$$

Where t_D is:

$$t_D = 2.9922 \times 10^{-4} \frac{t}{w} \sqrt{\frac{\sqrt{k_x k_z}\, g \alpha}{m \upsilon_{os} \phi \Delta s_o h}} \tag{29}$$

As it is described before all the variables are in SI unit, and for the sake of more comfort υ_{os} is in cp.m^3/kg. The parameter t_D is similar to that was obtained by Butler et al (1980) with a little bit difference, inserting reservoir half-width in lieu of reservoir height in the denominator of the time fraction. These dimensionless groups are designated in a novel way so that we could portray each side of the reservoir like a square with aspects of unity. It could also help us for calculating the recovery factor by tracking down the interface into the reservoir during its period of progress.

Hereby the equation 27 may also be represented in dimensionless form:

$$Y = 1 - \frac{1}{4}\left(\frac{t_D^*}{X}\right)^2 \qquad (30)$$

Values of Y calculated from equation 30 have been plotted against X in figure 4. Note that in figure 4 when time increases, the steam-oil interface moves away from the point $\left(0, Y_P\right)$ where the horizontal producer well is located. The steam zone in the figure becomes larger as oil drains by gravity out of the system. Eventually, after a long period of time, the reservoir has been depleted of oil by gravity drainage and only a steam zone above the producer exists. It is easily obvious that not whole the reservoir could be produced via SAGD but up to the depth "Y_p" may be produced with the help of steam-assisted gravity drainage. The region below the horizontal producer that cannot be produced is shown in figures 4 and 5 in a blue-bricked pattern. This assumption would be reasonable in comparison with Butler and Stephens (1980) that they located the horizontal producer in the bottom section of the reservoir at absolute zero ordinate but as it is clear in very rare situations a horizontal producer could be drilled right at the origin. Therefore it calls for a modification to the assumption they used.

Figure 4 Interface Curves- proposed scheme.

2.4 Steam-Oil Interface Positioning - Butler Theory along with TANDRAIN

Butler et al (1981) introduced a formula in dimensionless form like that of equation 30 in this form:

$$Y = 1 - \frac{1}{2}\left(\frac{t'_D}{X'}\right)^2 \tag{31}$$

In this equation Y is the same as defined in the pervious section, but t'_D and X' differ a little bit. Also, the fraction $\frac{t'_D}{X'}$ is indeed the same as the fraction $\frac{t_D}{X}$ with the dimensionless variables defined previously. They are:

$$t'_D = 2.9922 \times 10^{-4}\, \frac{t}{h}\sqrt{\frac{\sqrt{k_x k_z}\, g\alpha}{m\,\upsilon_{os}\,\phi \Delta s_o h}} \tag{32}$$

$$X' = \frac{x}{h} \tag{33}$$

In the interim, the basic SAGD analytical expression does not take into account how the heated oil flows horizontally to the horizontal producer as the oil-steam interface moves away horizontally from the point (*0*, Y_p). In reality, the oil-steam interface will frequently stay at the horizontal production well as the steam zone grows larger above the well, rather than moving horizontally away from the horizontal well (Aguilera et al (1991)). It means that a modification to the previous works ought to be added and this modification is referred to as TANDRAIN (Butler and Stephens (1980)). Basically, what TANDRAIN does is draw a tangent line from the horizontal production well location to the steam-oil interface curves for particular points in time (Aguilera et al (1991)).

Figure 5 Interface Curve- TANDRAIN assumption

At point X_t two criteria must be satisfied:

$$\text{Criterion (1)}: \quad Y_{line} = mX_t + Y_p = 1 - \frac{t_D^2}{2} \cdot \frac{1}{X_t^{\,2}} \tag{34}$$

$$\text{Criterion (2)}: \quad m = \frac{dY}{dX}\Big|_{X_t} \tag{35}$$

Solving these two equations simultaneously we may have:

$$X_t = t_D \sqrt{\frac{3}{2(1-Y_p)}} \quad , \quad m = \frac{\sqrt{8(1-Y_p)^3}}{3t_D\sqrt{3}}$$

If $X_t < 1 \;\Rightarrow\; t_D < \sqrt{\frac{2(1-Y_p)}{3}}$ and hence:

$$Y = \left(\frac{\sqrt{8(1-Y_p)^3}}{3t_D\sqrt{3}} \right) X + Y_p \qquad \text{if } X < t_D \sqrt{\frac{3}{2(1-Y_p)}} \tag{36}$$

$$Y = 1 - \frac{t_D^2}{2} \cdot \frac{1}{X^2} \qquad \text{if } X \geq t_D \sqrt{\frac{3}{2(1-Y_p)}} \tag{37}$$

And if $X_t \geq 1 \Rightarrow t_D \geq \sqrt{\frac{2(1-Y_p)}{3}}$:

$$Y = \left(\frac{\sqrt{8(1-Y_p)^3}}{3t_D\sqrt{3}} \right) X + Y_p \tag{38}$$

Finally, the interface position into the half-width of reservoir based on Butler and Stephens' formulation (Butler et al (1981)) in the most general form is as the following:

$$Y = \left[\left(\frac{\sqrt{8(1-Y_p)^3}}{3t_D\sqrt{3}} \right) X + Y_p \right] \left(1 - U\left(X - t_D \sqrt{\frac{3}{2(1-Y_p)}} \right) \right) + \left(1 - \frac{t_D^2}{2} \cdot \frac{1}{X^2} \right) U\left(X - t_D \sqrt{\frac{3}{2(1-Y_p)}} \right) \tag{39}$$

Regarding equation 38, values of Y have been plotted against X in figure 6.

Figure 6 Interface Curves-based on Butler et al (1981)

As we can see in figures 4 and 6 the interface of Butler et al (1981) and those which are obtained in this work are similar in behavior, however they differ quantitatively. The precision of the proposed theory is to be examined below with the help of recovery factor matching.

2.5 Recovery Factor determination - Butler Theory along with TANDRAIN

For comparison purposes, the recovery factor of Butler and Stephens' (1980) and that of the proposed scheme have been calculated. They were compared with each other to provide us judgment about the accuracy of the method. By applying the new defined dimensionless groups and taking advantage of TANDRAIN phenomenon the recovery factor of Butler et al (1981) could be expressed as:

$$Y_p = 1 - \frac{t_D'^2}{2X'^2} = 1 - \frac{t_D^2}{2X^2},$$

$$X_t = t_D \sqrt{\frac{3}{2(1-Y_p)}} \Rightarrow RF = 1 - \left[Y_p.X_t + m.\frac{X_t^2}{2} + \int_{X_t}^{1} \left(1 - \frac{t_D^2}{2X^2} \right) dX \right]$$

$$RF_{B\_TANDRAIN} = t_D \left(\sqrt{\frac{3}{2}(1-Y_p)} - \frac{t_D}{2} \right) \left(1 - u\left(t_D - \sqrt{\frac{2(1-Y_p)}{3}} \right) \right) + \left(1 - Y_p - \frac{1}{3\ t_D} \sqrt{\frac{2(1-Y_p)^3}{3}} \right) u\left(t_D - \sqrt{\frac{2(1-Y_p)}{3}} \right) \quad (40)$$

As it is clear the recovery factor here based on the Butler and Stephens (1980) theorem is not a straight line with a constant slope but a parabola. Meanwhile, in figure 7 it can be seen that while the location of horizontal producer varies the ultimate recovery varies consequently. It means that by varying the location of producer on the vertical axis, the recovery factor varies thereafter. It clearly seems logical because the production mechanism in SAGD is just due to gravity drainage. Therefore, it looks to be necessary to include the location of horizontal producer in the analytical modeling of SAGD. Note that the equation 40 has been derived as a consequence of defining new dimensionless groups (scaling the reservoir dimensions into the range of 0 to 1) and establishing the drained area between two consecutive time steps.

Figure 7 the effect of Horizontal Producer location on RF- based on Butler and Stephens (1980)

In this figure, it can bee seen that the higher the location of horizontal producer, the less the value of ultimate recovery would be. It seems quite reasonable why the height of oil column above the production well decreases and consequently the gravity forces diminishes somewhat.

2.6 Steam-Oil Interface Positioning – Proposed scheme along with TANDRAIN

The TANDRAIN modification along with the new formulation gives:

From equations 27 we have;

$$y = h - \left(2.9922 \times 10^{-4} \frac{q_D}{2} \right)^2 \frac{\sqrt{k_x k_z}\, g\alpha}{m \upsilon_{os} \phi \Delta s_o} \left(\frac{t}{x} \right)^2 \quad (27)$$

$$\frac{y}{h}=1-\left(2.9922\times10^{-4}\,\frac{q_D}{2}\right)^2\frac{\sqrt{k_x k_z}\,g\alpha}{m\upsilon_{os}\phi\Delta s_o h}\left(\frac{t}{x}\right)^2 \tag{41}$$

Like it is done based on Butler's (1981) at point X_t two criteria must be satisfied:

$$\text{Criterion (1)}: \quad Y_{line}=mX_t+Y_p=1-\frac{t_D^{*\,2}}{4}\cdot\frac{1}{X_t^{\,2}} \tag{42}$$

$$\text{Criterion (2)}: \quad m=\left.\frac{dY}{dX}\right|_{X_t} \tag{43}$$

Solving these two equations simultaneously we may have:

$$X_t=\frac{t_D^*}{2}\sqrt{\frac{3}{1-Y_p}} \tag{44}$$

$$m=\frac{4(1-Y_p)}{3t_D^*}\sqrt{\frac{1-Y_p}{3}} \tag{45}$$

If $X_t<1 \;\Rightarrow\; t_D^*<2\sqrt{\frac{1-Y_p}{3}}$:

$$Y=\left(\frac{4}{3t_D^*}\sqrt{\frac{(1-Y_p)^3}{3}}\right)X+Y_p \qquad \text{if } X<\frac{t_D^*}{2}\sqrt{\frac{3}{1-Y_p}} \tag{46}$$

$$Y=1-\frac{t_D^{*\,2}}{4}\cdot\frac{1}{X^2} \qquad \text{if } X\geq\frac{t_D^*}{2}\sqrt{\frac{3}{1-Y_p}} \tag{47}$$

Where: $t_D^*=q_D\,t_D$

And If $X_t\geq1 \;\Rightarrow\; t_D^*\geq2\sqrt{\frac{1-Y_p}{3}}$:

$$Y=\left(\frac{4}{3t_D^*}\sqrt{\frac{(1-Y_p)^3}{3}}\right)X+Y_p \tag{48}$$

Finally, the interface position into the half-width of reservoir in the most general form is as the following:

$$\begin{aligned}Y=&\left[\left(\frac{4}{3t_D^*}\sqrt{\frac{(1-Y_p)^3}{3}}\right)X+Y_p\right]\left(1-u\left(X-\frac{t_D^*}{2}\sqrt{\frac{3}{1-Y_p}}\right)\right)\\&+\left(1-\frac{t_D^{*\,2}}{4}\cdot\frac{1}{X^2}\right)u\left(X-\frac{t_D^*}{2}\sqrt{\frac{3}{1-Y_p}}\right)\end{aligned} \tag{49}$$

This equation may be put into an equation with the dimensionless time similar to that of Butler and Stephens (1980). It gives:

$$Y = \left[\left(\frac{4}{3q_D\,t_D}\sqrt{\frac{(1-Y_p)^3}{3}}\right)X + Y_p\right]\left(1-u\left(X-\frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}}\right)\right) + \left(1-\frac{(q_D\,t_D)^2}{4}.\frac{1}{X^2}\right)u\left(X-\frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}}\right) \tag{50}$$

The function $u\left(X-\frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}}\right)$ is acting as a step function with this definition:

$$u\left(X-\frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}}\right) = \begin{cases} 0 \;\; if \;\; X \prec \frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}} \\ 1 \;\; if \;\; X \geq \frac{q_D\,t_D}{2}\sqrt{\frac{3}{1-Y_p}} \end{cases} \tag{51}$$

Now we are seeking for estimating the fraction of the original oil in place that has been produced due to steam-assisted gravity drainage. Knowing the location of interface at any particular point in time, we may easily calculate the area that has been drained. Subtracting this calculated drained area from the displaceable area could lead us to recovery factor up to that particular time. The described process for recovery calculation has been done in another way by Butler et al (1981). They did the calculation by means of a numerical method by combining equations 29 and 33. The proposed equation was as below:

$$\delta X_i = (1-X_{n-1})\left(\sqrt{n-i}-\sqrt{n-i-1}\right) \tag{52}$$

This equation is used repetitively to calculate successive positions of the interface (Butler et al (1980)). Also *n* denotes the index of each stage of calculations. However in this work it has mentioned that it is possible for suggesting an explicit-analytical-method stands for the recovery calculations precisely.

2.7 Recovery Factor determination- Proposed scheme

Since the cumulative recovery factor could be connected directly to the progress of interface within the reservoir, it could be formulated based on a simple frame as below:

$$RF = Displceable\;\; Area - Area\;\; right\;\; now \tag{53}$$

The "*area right now*" could be obtained by establishing the area under the interface curve at any particular time. Since the half-width reservoir has been scaled as a unit aspect square and its total area is 1, the displaceable area will be $1\text{-}Y_p$. Provided that:

$$RF = 1 - Y_p - \left(Y_p X_t + mX_t \times \frac{X_t}{2} + \int_{X_t}^{1} \left(1 - (\frac{t_D^{*\,2}}{4}) \frac{1}{X^2} \right) dX \right) \tag{54}$$

Also note that while using this formula the recovery factors are to be obtained based on the recoverable oil. It means that the calculated recovery factors according to the equation 52 must be multiplied by $1 - s_{org}$ in case the residual oil saturation for gas injection is nonzero. This point should be well considered all over this paper.

If $X_t < 1 \;\Rightarrow\; t_D^* < 2\sqrt{\frac{1-Y_p}{3}}$:

$$RF = \frac{t_D^*}{2}\left(\sqrt{3(1-Y_p)} - \frac{t_D^*}{2} \right) \qquad \text{if } t_D^* < 2\sqrt{\frac{1-Y_p}{3}} \tag{55}$$

Where: $t_D^* = q_D\, t_D$

And If $X_t \geq 1 \;\Rightarrow\; t_D^* \geq 2\sqrt{\frac{1-Y_p}{3}}$:

$$RF = 1 - Y_p - \left(\frac{2}{3t_D^*}\sqrt{\frac{(1-Y_p)^3}{3}} \right) \qquad \text{if } t_D^* \geq 2\sqrt{\frac{1-Y_p}{3}} \tag{56}$$

The recovery factor could be also presented in the most general form:

$$RF = \left(\begin{array}{l} \frac{t_D^*}{2}\left(\sqrt{3(1-Y_p)} - \frac{t_D^*}{2} \right) \left(1 - u\left(t_D^* - 2\sqrt{\frac{1-Y_p}{3}} \right) \right) \\ + \left(1 - Y_p - \left(\frac{2}{3t_D^*}\sqrt{\frac{(1-Y_p)^3}{3}} \right) \right) u\left(t_D^* - 2\sqrt{\frac{1-Y_p}{3}} \right) \end{array} \right) (1 - s_{org}) \tag{57}$$

and in the form with similar dimensionless time to Butler's it could be expressed as:

$$RF = \left(\begin{array}{l} \frac{q_D\, t_D}{2}\left(\sqrt{3(1-Y_p)} - \frac{q_D\, t_D}{2} \right) \left(1 - u\left(q_D t_D - 2\sqrt{\frac{1-Y_p}{3}} \right) \right) \\ + \left(1 - Y_p - \left(\frac{2}{3q_D\, t_D}\sqrt{\frac{(1-Y_p)^3}{3}} \right) \right) u\left(q_D t_D - 2\sqrt{\frac{1-Y_p}{3}} \right) \end{array} \right) (1 - s_{org}) \tag{58}$$

In the figure below it is easily visible that the more the value of Y_p, the less the ultimate recovery. It also depicts the connection between the main dimensionless groups which have been described before; they are dimensionless time (t_D), dimensionless rate parameter (q_D), and dimensionless producer location (Y_p), as well as recovery factor (RF).

Figure 8 the effect of Horizontal Producer location on RF- based on "New Method"

In the meantime, a set of calculated interface curves is given in figure 9 that depicts the position of interface at different dimensionless time.

Figure 9 Interface curves with TANDRAIN assumption-New Theory (Half-width Reservoir)

It is also possible to locate the interface in the whole reservoir. Figure 10 portrays the location of oil-steam interface within a heavy oil reservoir over a long period of time in dimensionless scale. As it is clear, having been used the TANDRAIN theorem (Butler and Stephens (1980)), interface is fixed at the horizontal production well at all the times.

Figure 10 Interface curves with TANDRAIN assumption-New Theory (Full-width Reservoir)

A comparison has been made among the results of new formulation in this work with those of Butler et al (1980, 1981) and also simulation results. It has been done in the following.

3. Simulation Model Description

The model used to obtain simulation results was half of a box-shaped reservoir with a drainage area of 7.5 acres and a constant thickness of 50 m. The porous medium has a homogenous porosity of 0.33, allowing areal permeability isotropy and vertical anisotropy with values in x, y, and z directions of 2000, 2000, and 800 md, respectively. Two horizontal well of radius 0.0875 m are located one above another in the lower part of the reservoir, spaced vertically 12 m apart from each other. They are centered at mid-width and completed wholly along the reservoir. Initially, there are two phases: water at an immobile saturation of 0.2, and oil with a high viscosity of 10000 cp. Capillary pressure effect is ignored. The effect of condensation over the interface, for the sake of convenience, is ignored as well. It means that the dimensionless temperature (θ_i) at the interface ($\zeta = 0$) is always equal to 1, same as all other previous works.

3.1 Simulator

The simulator used in this study is a three phase Thermal simulator. It allows an adaptive implicit-explicit grid formulation. This formulation reduces computer execution time by applying an IMPES type solution to certain grid blocks that do not need to be solved fully implicitly.

3.2 Grid Selection

In the numerical study of steam-assisted gravity drainage in a heavy oil reservoir it is usual to simulate just one side of the reservoir and considering symmetry. Also, it is important to ensure that the simulator grid block sizes do not influence the performance results. Unless a proper griding system is obtained, we may incur the fluctuation in oil rate and underestimate the recovery factor due to temperature dispersion over the grid block volume. Since it takes much more time to heat a larger grid block to oil mobilization temperature, fluctuation in oil rate and underestimation in recovery factor may be encountered.

To study the sensitivity of simulation results to grid size, simulation runs were made at similar conditions and the results were then plotted versus grid sizes and an appropriate grid block size is selected at the point where performance results converge as grid block size becomes smaller (Figure 11).

Fluid flow is expected to be fast and radial near the well bore. For this reason, Cartesian grid blocks should be small enough for high flow resolution and equally sized for better accuracy. With increasing distance from the well, flow properties change less rapidly. In this case, grid blocks may become large in order to save computer time and storage. With this in mind, four grids have been simulated: Uniform Coarse Grid with 135 blocks, Uniform Fine Grid with 23595 blocks, Non-Uniform Fine Grid with 16830 blocks, and Non-Uniform Medium Grid with 1309 blocks.

Figure 11-Influence of Grid System on simulation results

According to Figure 11 the Uniform Fine Griding is more than acceptable for representing an element of symmetry to the SAGD process.

4. “New Method” versus “Butler et al (1980, 1981)”

After being described the new formulations in detail and being done a modification to the definition of dimensionless variables of Butler at al (1980, 1981), some comparisons among the precision of “New Method” and “Butler’s” as well as simulation results have been made.

Since the effect of condensation over the interface is overlooked, the value of θ_i^m would be equal to 1 all the times. Hence, the equation 59 that is based on recoverable oil (disregarding the effect of residual oil saturation for gas injection) could be rearranged in this way:

$$\begin{aligned} RF = {} & \frac{q'_D\, t_D}{2}\left(\sqrt{3\ k_{ro} N(1-Y_p)} - \frac{q'_D\, t_D\, k_{ro} N}{2}\right)\left(1-u\left(q'_D t_D - 2\sqrt{\frac{1-Y_p}{3\ k_{ro} N}}\right)\right) \\ & + \left(1-Y_p-\left(\frac{2}{3\ q'_D\, t_D}\sqrt{\frac{(1-Y_p)^3}{3\ k_{ro} N}}\right)\right) u\left(q'_D t_D - 2\sqrt{\frac{1-Y_p}{3\ k_{ro} N}}\right) \end{aligned} \qquad (59)$$

Where,

$$q'_D = \sqrt{\frac{m}{2^{m-1}}\int_0^{\infty}\left[e^{-N\varsigma}.erfc\left(\frac{\varsigma}{2\sqrt{\tau}} - \frac{N\sqrt{\tau}}{2}\right) + erfc\left(\frac{\varsigma}{2\sqrt{\tau}} + \frac{N\sqrt{\tau}}{2}\right)\right]^m d\varsigma} \qquad (60)$$

In the above equation there are two parameters which are quite ambiguous and there is no any straight method stands for obtaining them explicitly. Also, despite the idea of Butler and Stephens (1980) that considered k_{ro} being normally about 0.4, we still have to obtain the value of N. Of course it would be very worthwhile to propose a method to calculate N because doing that, we can obtain the value of the interface velocity- a parameter which has not been formulated or estimated as of yet.

All in all, from equation 58 it is clear that N should be calculated to make the SAGD process completely clear in a heavy oil reservoir, although due to the lack of an explicit relationship to do so we may have to use the theory of type curves. In the figure below, we see a set of type-curves depicts the relationship among dimensionless time, and recovery factor as well as N for a particular amount of y_p .

Figure 12-New Type-Curves for Recovery Factor versus dimensionless group- yp=0.04

This is a set of type-curves by which we may obtain a rough estimation of N that would be a good representative for average interface velocity in the duration of SAGD process.

In figure below an attempt of matching the simulation results over the type-curves has been done. It illustrates a comparatively good match between the simulation data points and those of curves related to N equal to 0.015 or 0.035.

Figure 13- setting up matches between Simulation Results and Type-Curves

Figure 14 compares the recovery factor calculated from the “New Method” by allocating the parameter N equal to 0.025, 0.035, and 1 with those calculated from Butler using the TANDRAIN assumption and the recovery factor obtained from the Thermal Simulator.

Figure 14 Cumulative drained Oil Recovery to horizontal producer

From this plot we can see that the proposed formulation (equation 57) works well in general, however that of Butler and Stephens (1980) overvalues the recovery factors together with drainage rates. According to this figure, at early times the “New Method” (like Butler and Stephens (1980)) overestimates the recovery factor via SAGD, however at late time it shows a good match among the curves and simulation trend. While in the “New Method” and any other researches, so far, the effect of heat transfer via the overburden and underburden has been ignored and also the effect of heat loss due to steam condensation over the interface has been overlooked, it would be quite reasonable to arrive at such consequences.

Besides, it is easily visible in this figure that the value of N affects the recovery in SAGD to an upper limit and that is around 2 arises from this plot. Also in this figure it can be seen that Butler and Stephens’ (1980) could lead us to results near to those of “New Method” with N equal to 2. It means that applying the “New Method” with high values of N (greater than 1) acts as if we have nearly applied the formulas suggested by Butler and Stephens (1980). Regarding the simulation model described before, it is clear in the figure below that the values of N greater than 1 do not affect the SAGD recovery and oil production rates. For this, if some operational parameters are subjected to change beyond a certain limit, the oil production and steam chamber sustainability will not be improved any more. For example the rate of steam injection or the injected steam temperature could be in any order, but more than a particular range nothing would be gained in case. Being obtained that range, extra expenses could be avoided. It’s a point which is ought to be concerned thoroughly while studying production optimization in SAGD.

Figure 15 “Interface Velocity Number” vs. Ultimate Recovery of simulation model

And in Figure 16 the conceivable values of interface velocity in this case is drawn versus ultimate recovery factor.

Figure 16 Interface Velocity vs. Ultimate Recovery of simulation model

According to this figure and figure 13 there is a way- proposed in this work -to get the interface velocity calculated which seems completely to be innovative.

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Nomenclature

k_x	Reservoir permeability in x direction (md)
k_z	Reservoir permeability in z direction (md)
g	Acceleration constant due to gravity (m/s^2)
α	Thermal diffusivity of reservoir (m^2/day)
φ	Porosity (dimensionless)
ΔS_o	Displaceable oil saturation (dimensionless)
ν_s	Cinematic oil viscosity at steam temperature (cp.m^3/kg)
μ	Oil viscosity (cp)
m	Viscosity constant (dimensionless)
h	Reservoir thickness (m)
y	The distance from the reservoir base (m)
w	Reservoir half-width (m)
ξ	New coordination variable (m)
ζ	Dimensionless distance in the new coordination
X	Dimensionless distance from the origin toward the x axis
Y	Dimensionless distance from the origin toward the vertical axis
Y_p	Dimensionless producer location
t	Time (day)
τ	Dimensionless time
t_D	Butler's dimensionless time
t_D^*	The proposed dimensionless time
u,U	Interface velocity (m/day)
N	Interface velocity number (dimensionless)
T	Temperature ahead of the moving interface (° K)
θ	Dimensionless temperature
θ_i	Dimensionless temperature at interface
s	Laplace transform variable
$\hat{\varphi}$	The angle between the producing element and the horizon
ρ_o	Oil density (kg/m^3)

ρ_g	Gas density (kg/m^3)
q	Oil production rate (m^3/day)
q_D	Dimensionless oil rate parameter
q'_D	Dimensionless oil rate parameter
RF	Recovery factor (fraction)
u	Step function

Figures Captions:

Figure 1. SAGD Principle, (courtesy of McDaniel)

Figure 2- Schema of SAGD process

Figure 3- SAGD production system

Figure 4- Interface Curves- proposed scheme.

Figure 5- Interface Curve- TANDRAIN assumption

Figure 6- Interface Curves-based on Butler et al (1981)

Figure 7- the effect of Horizontal Producer location on RF- based on Butler and Stephens (1980)

Figure 8- the effect of Horizontal Producer location on RF- based on "New Method"

Figure 9- Interface curves with TANDRAIN assumption-New Theory (Half-width Reservoir)

Figure 10- Interface curves with TANDRAIN assumption-New Theory (Full-width Reservoir)

Figure 11- Influence of Grid System on simulation results

Figure 12- New Type-Curves for Recovery Factor versus dimensionless group-yp=0.04

Figure 13- setting up matches between Simulation Results and Type-Curves

Figure 14- Cumulative drained Oil Recovery to horizontal producer

Figure 15- "Interface Velocity Number" vs. Ultimate Recovery of simulation model

Figure 16- Interface Velocity vs. Ultimate Recovery of simulation model

Figure 1

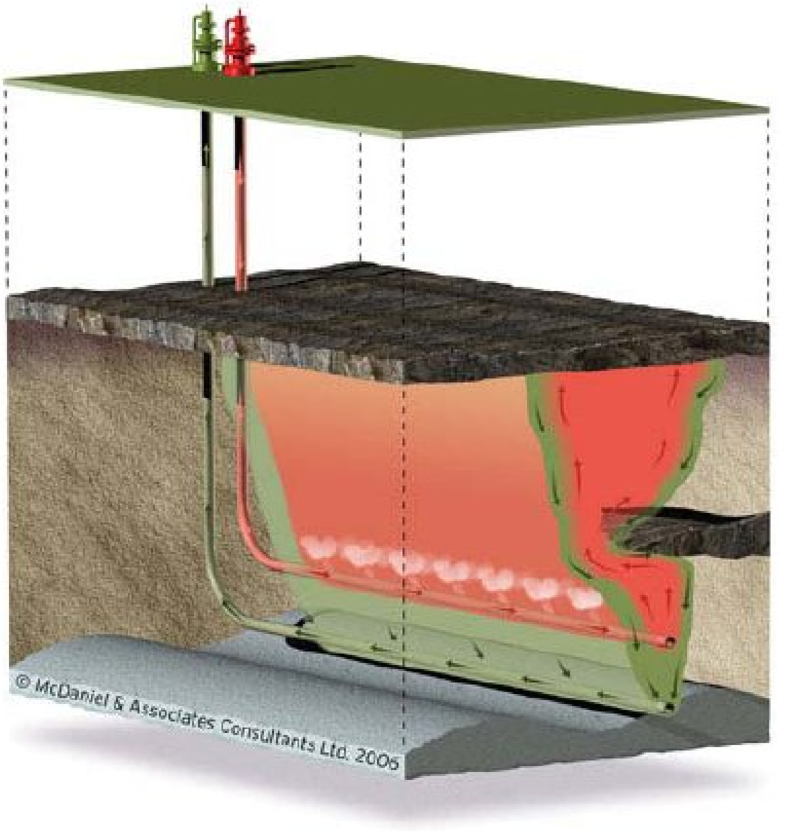

Figure 1. SAGD Principle, (courtesy of McDaniel)

Figure 2

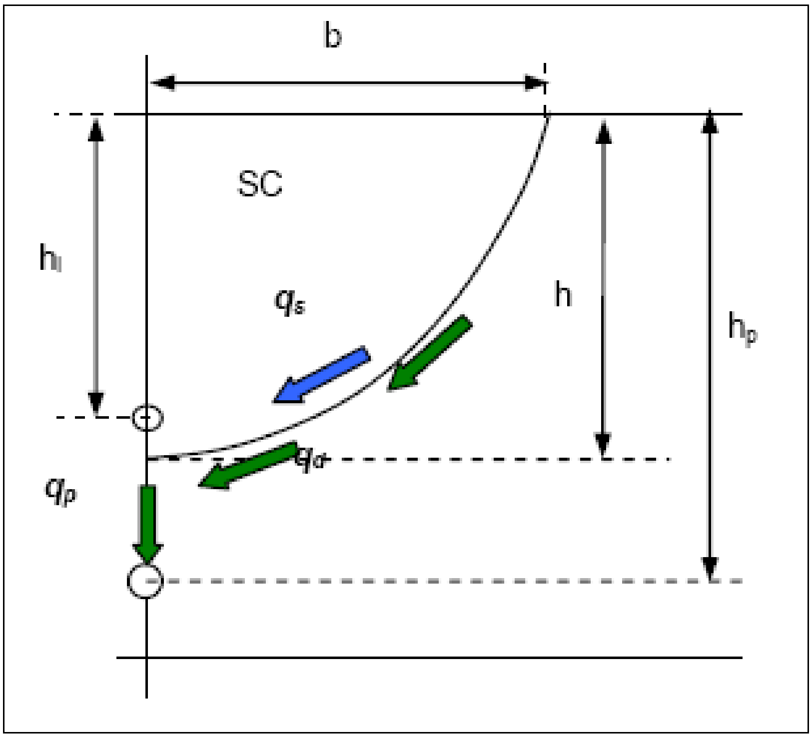

Figure 2. Schema of SAGD process

Figure 3

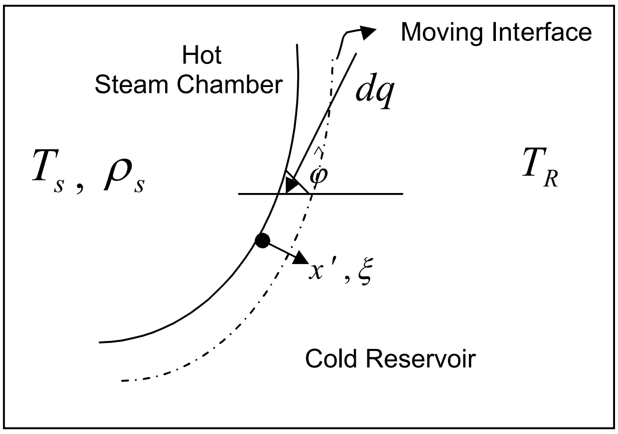

Figure 3. SAGD production system

Figure 4

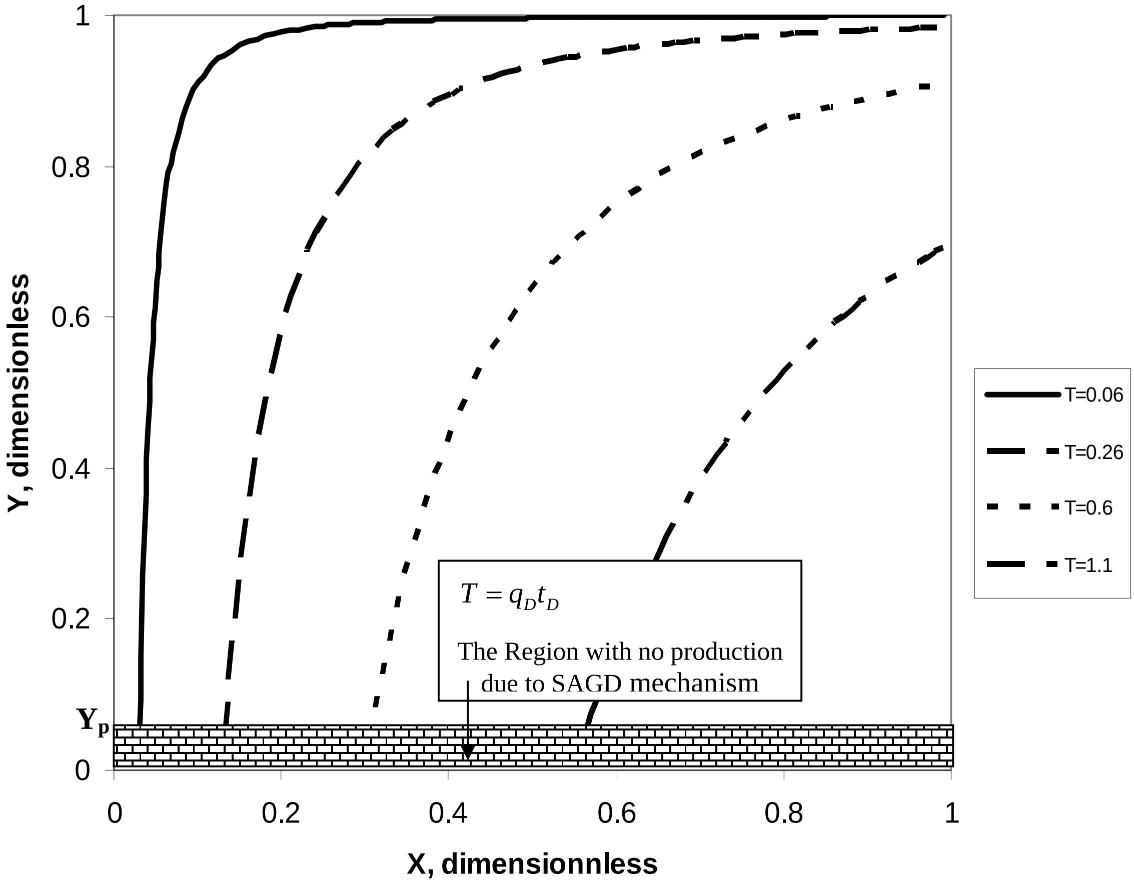

Figure 4. Interface Curves, proposed scheme.

Figure 5

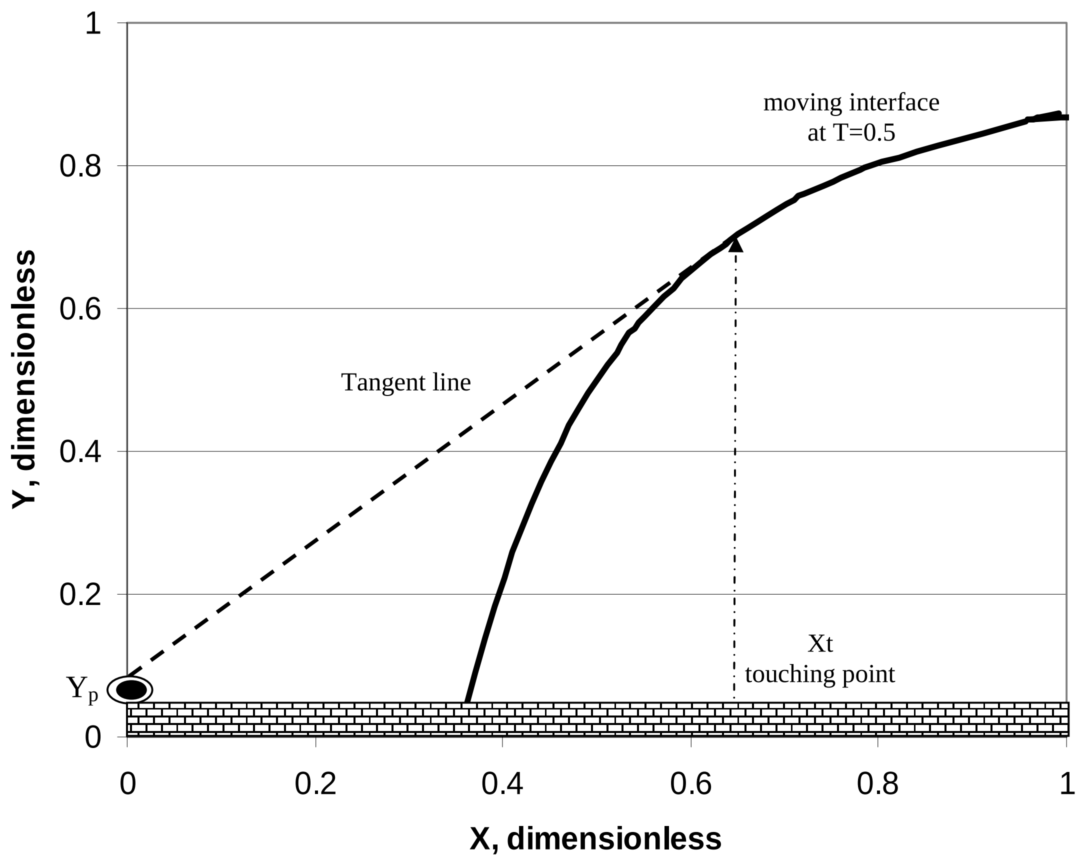

Figure 5. Interface Curves, TANDRAIN assumption.

Figure 6

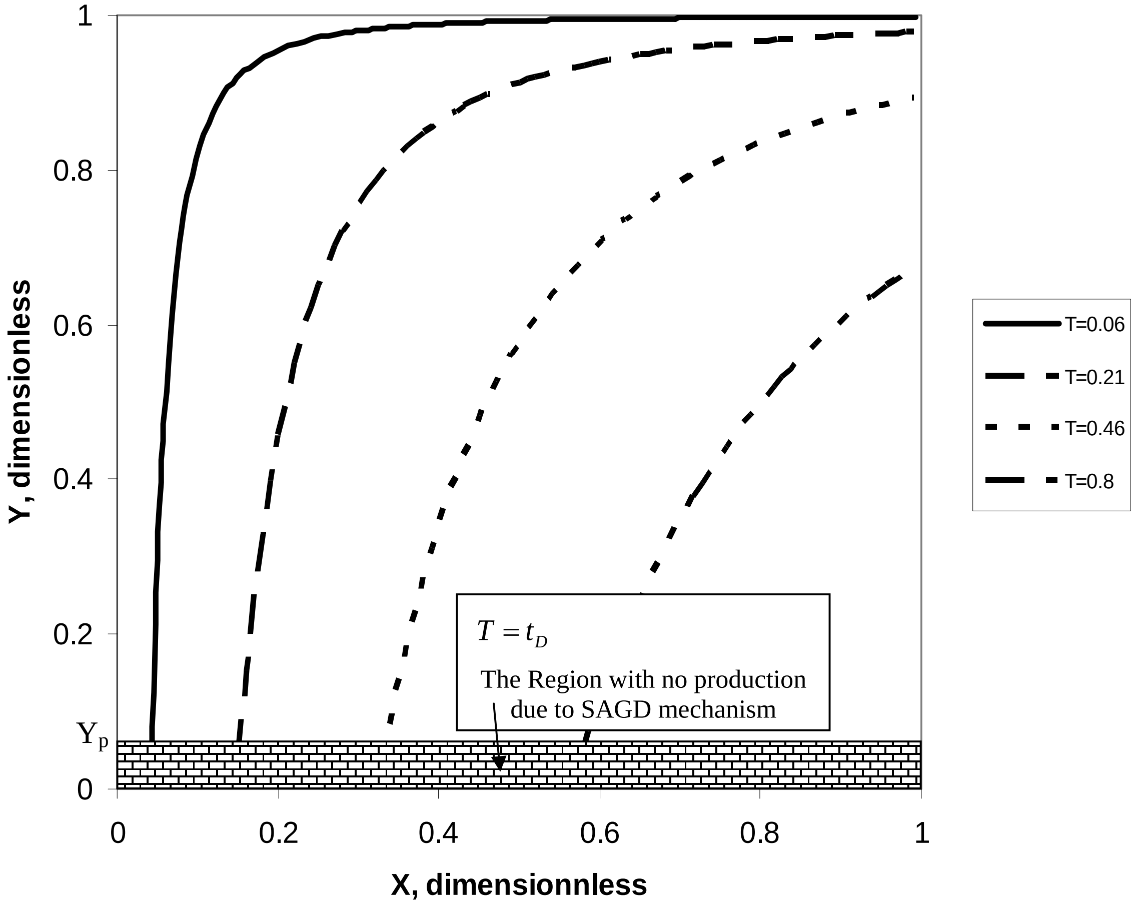

Figure 6. Interface Curves, Butler et al [2].

Figure 7

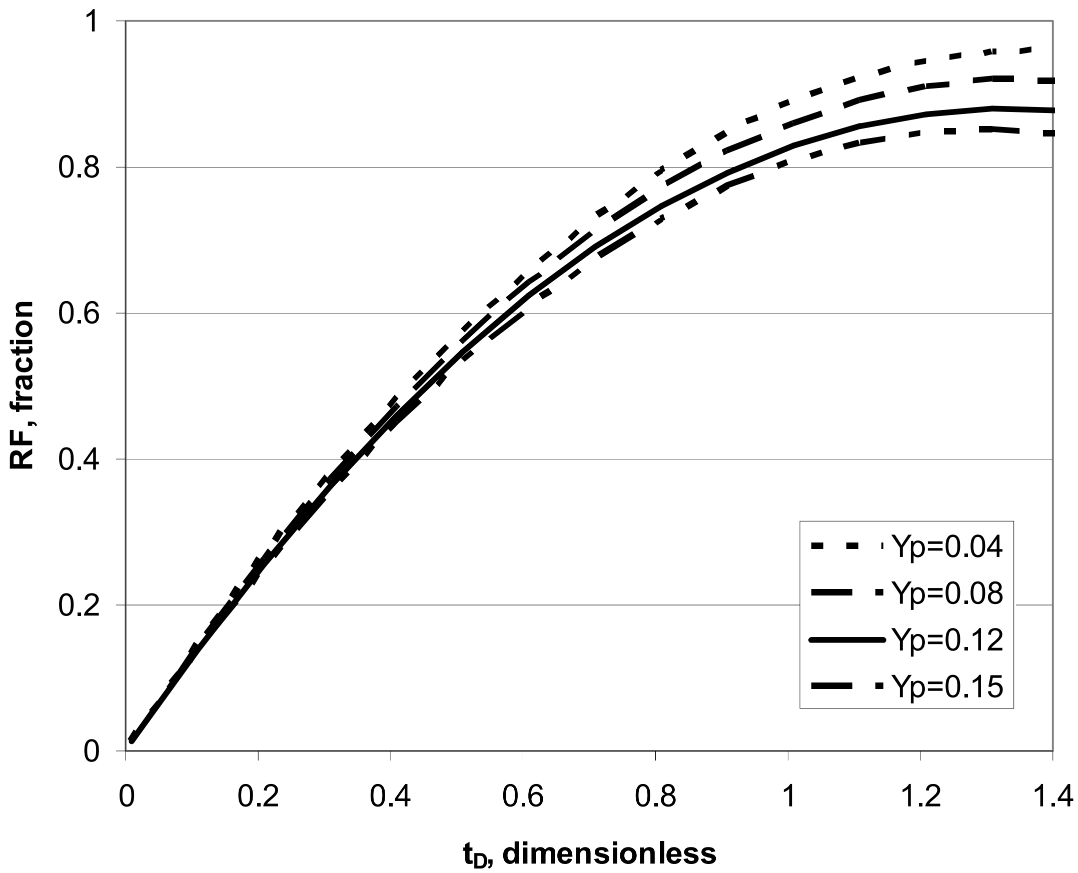

Figure 7. The effect of Horizontal Producer location on RF- Butler and Stephens [1].

Figure 8

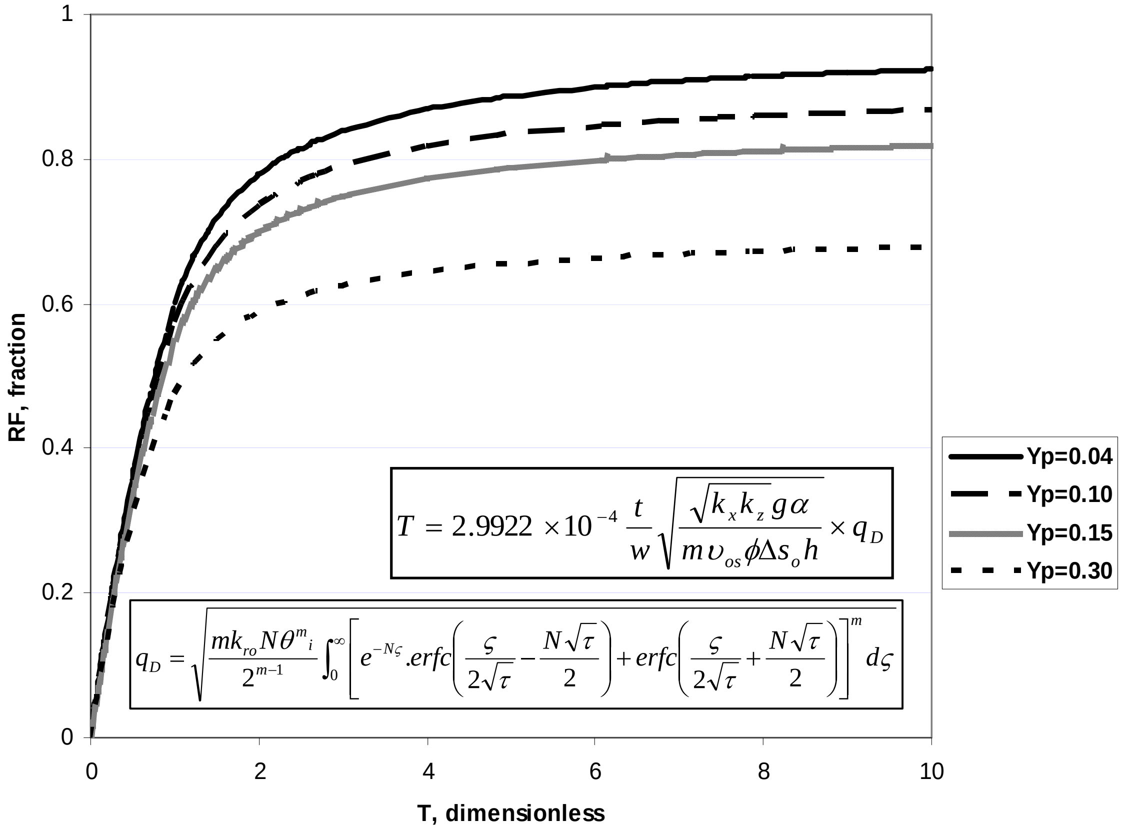

Figure 8 the effect of Horizontal Producer location on RF- based on “New Method”

Figure 9

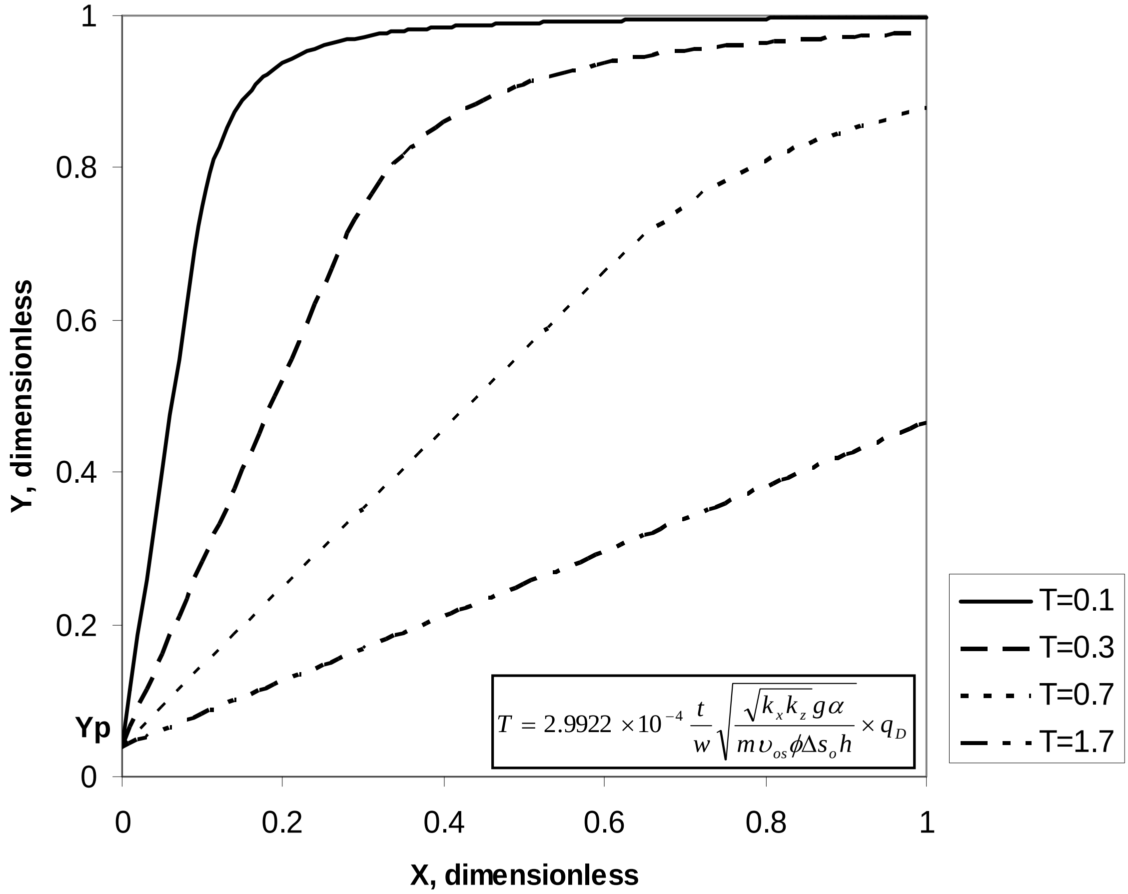

Figure 9 Interface curves with TANDRAIN assumption-New Theory (Half-width Reservoir)

Figure 10

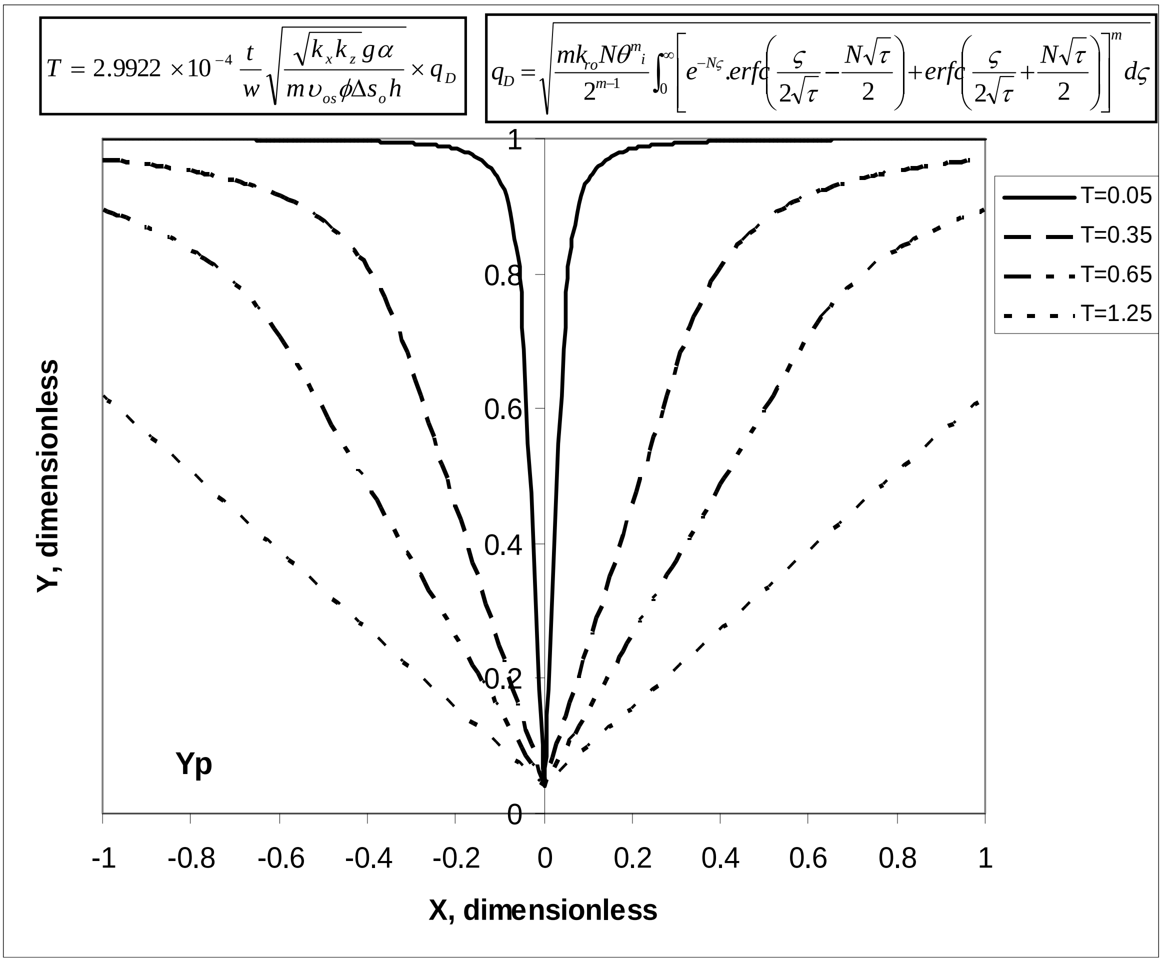

Figure 10 Interface curves with TANDRAIN assumption-New Theory (Full-width Reservoir)

Figure 11

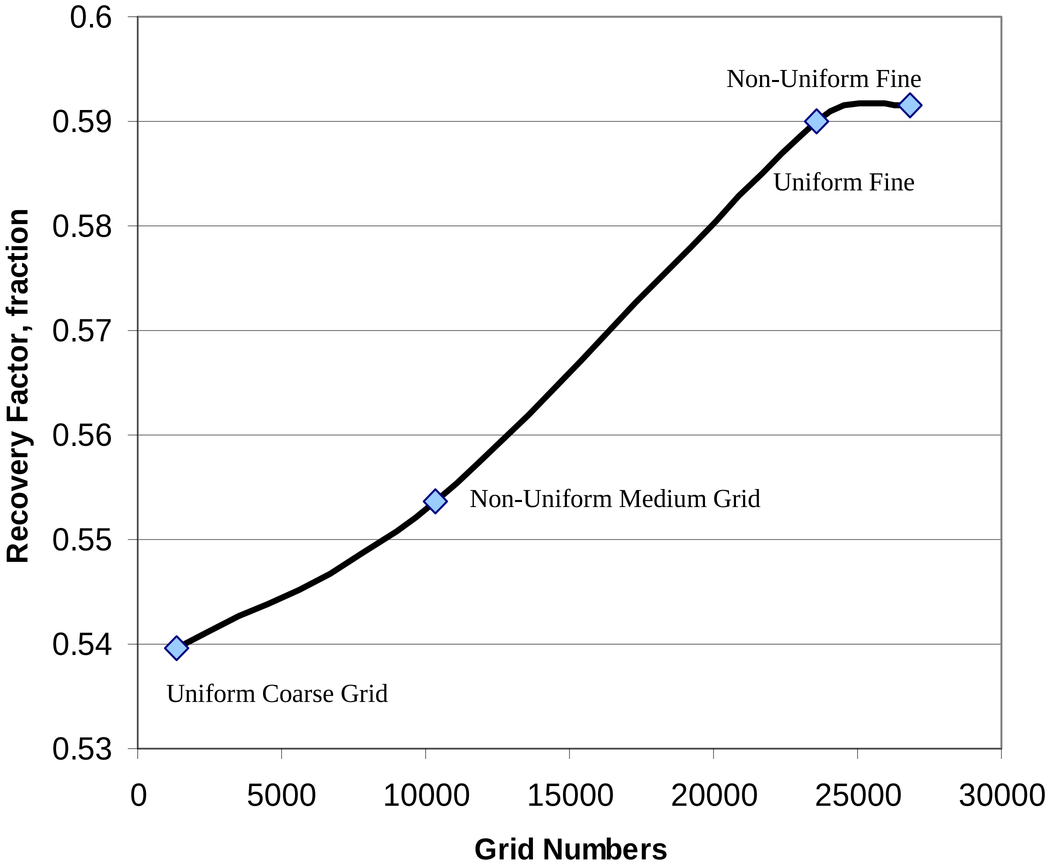

Figure 11-Influence of Grid System on simulation results

Figure 12

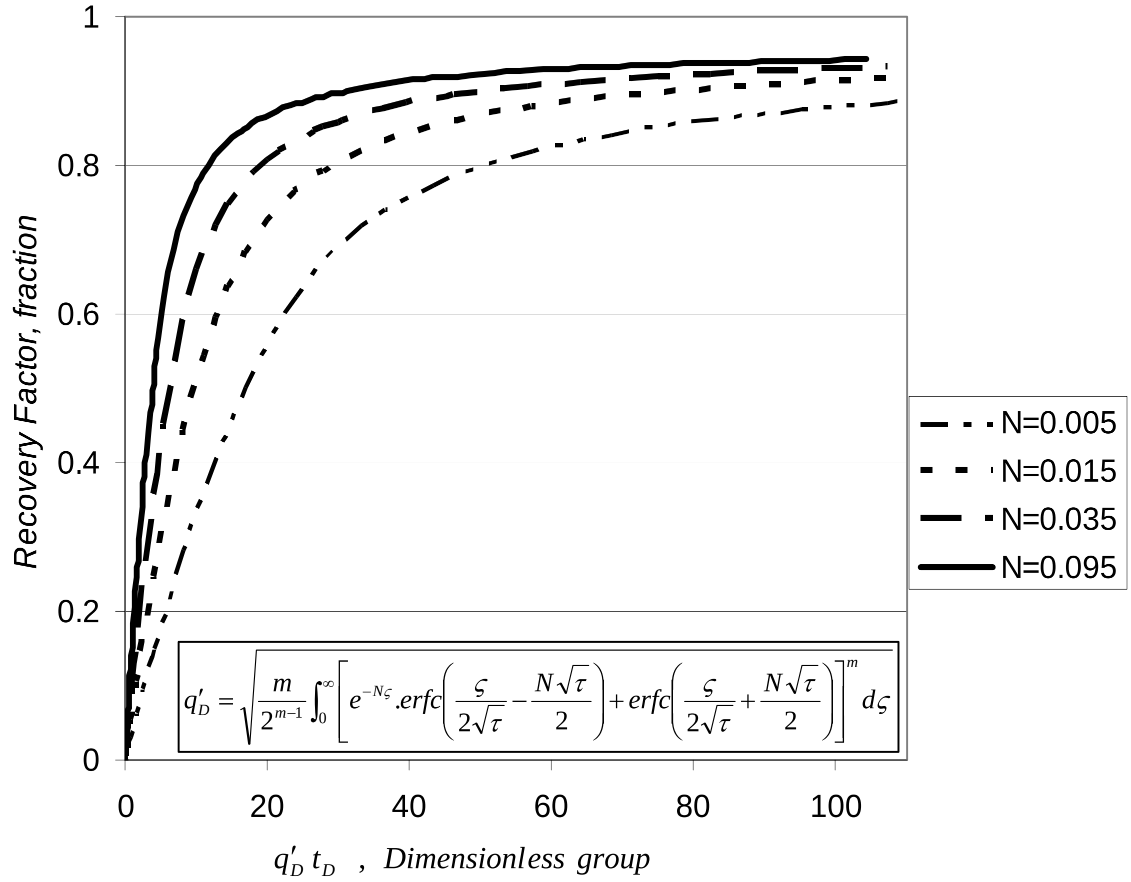

Figure 12-New Type-Curves for Recovery Factor versus dimensionless group- yp=0.04

Figure 13

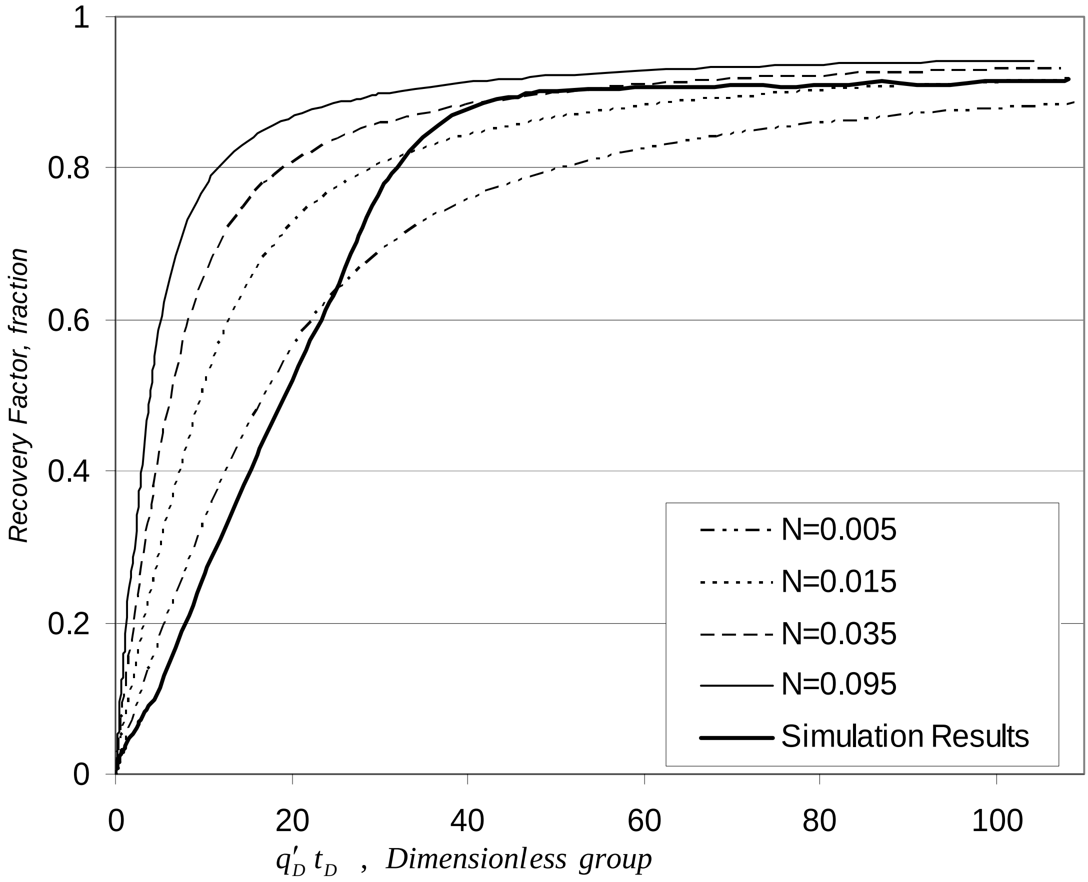

Figure 13- setting up matches between Simulation Results and Type-Curves

Figure 14

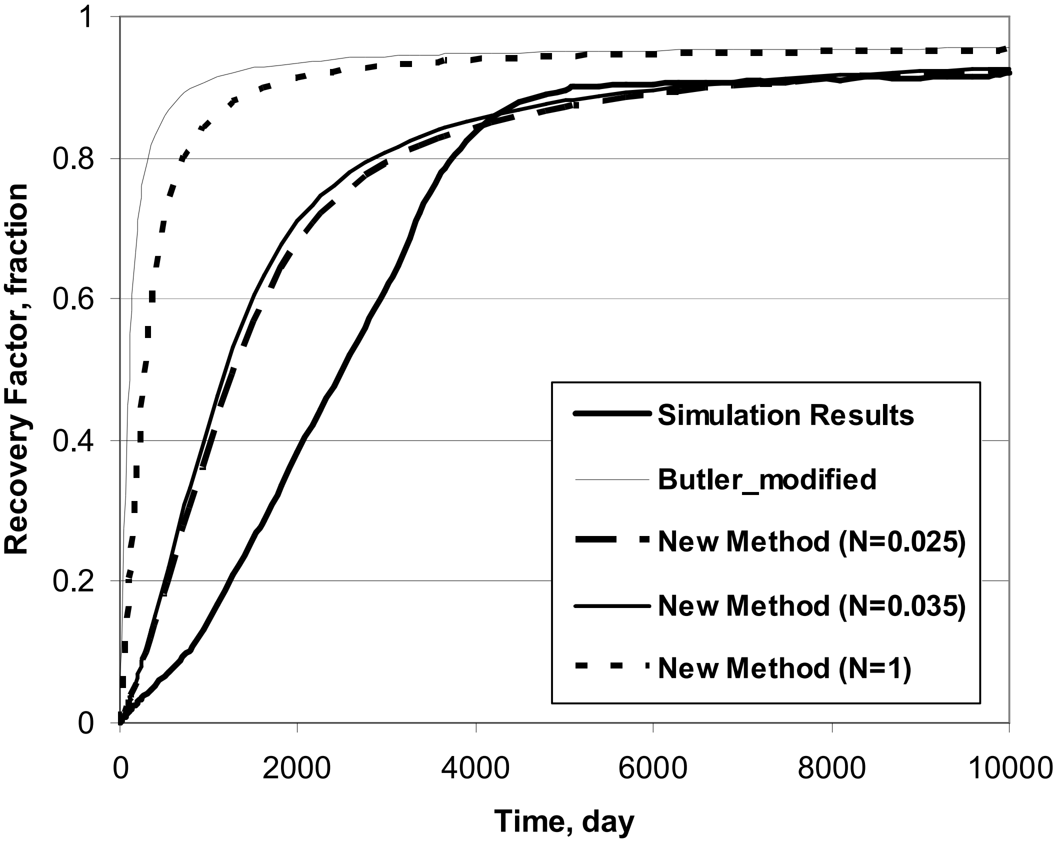

Figure 14 Cumulative drained Oil Recovery to horizontal producer

Figure 15

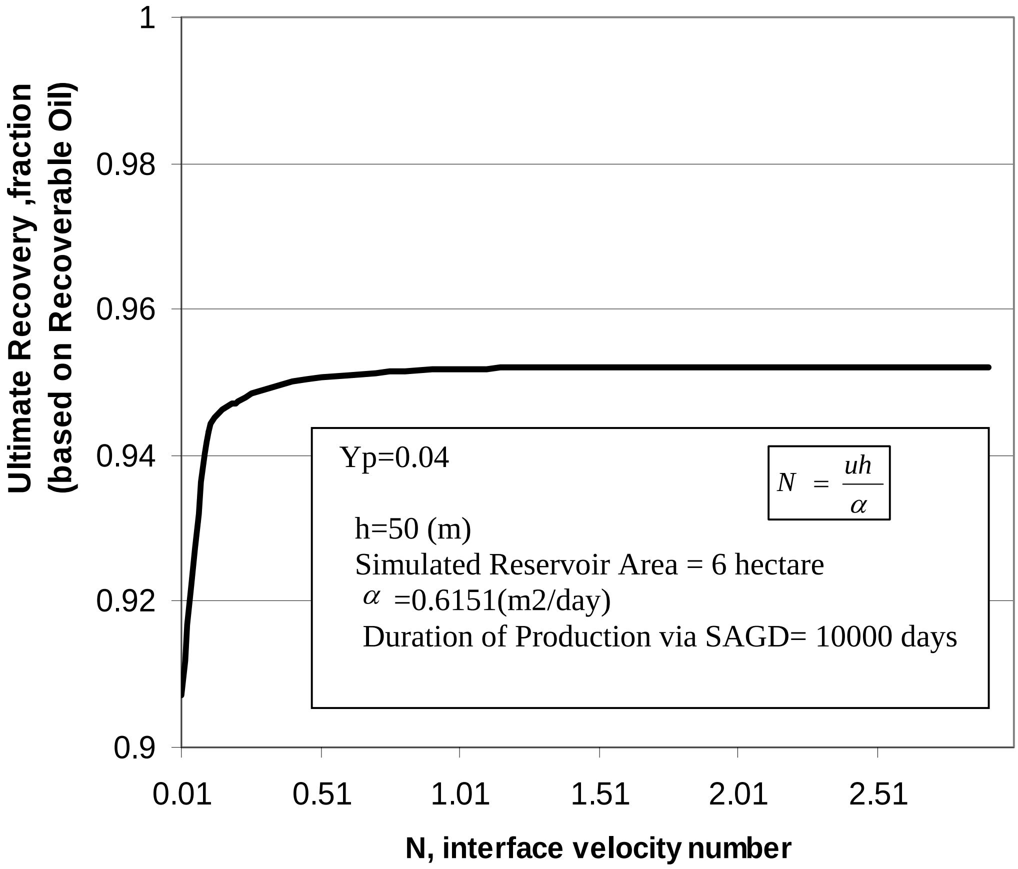

Figure 15 “Interface Velocity Number” vs. Ultimate Recovery of simulation model

Figure 16

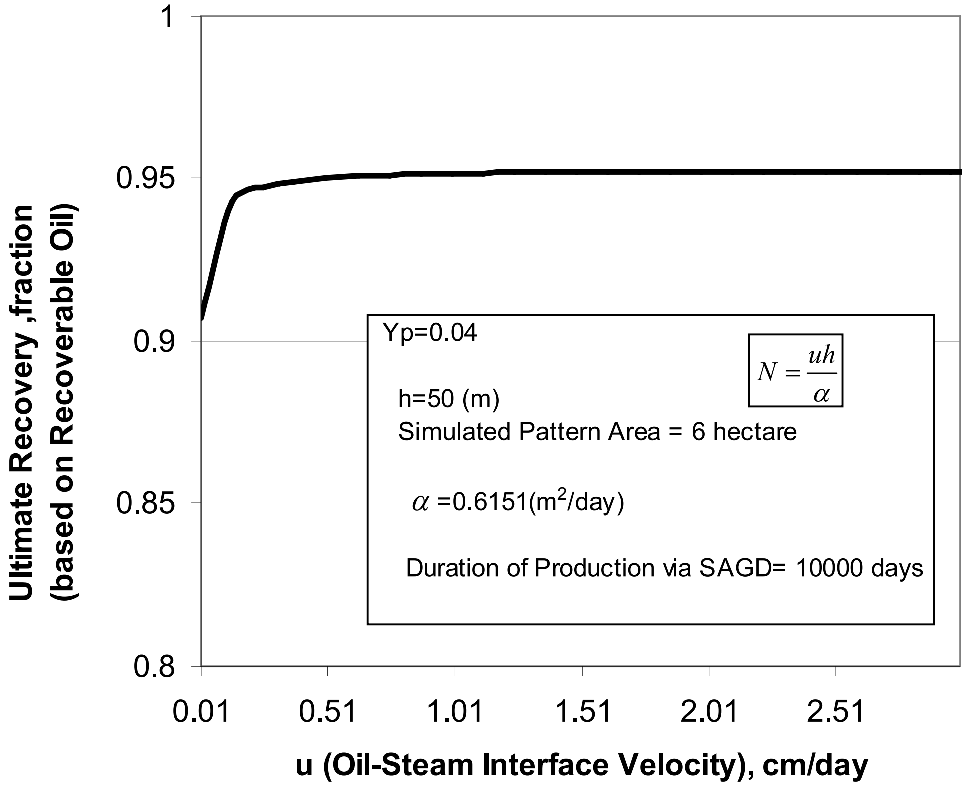

Figure 16 Interface Velocity vs. Ultimate Recovery of simulation model